SOLUTION OF A PROBLEM IN CURVES OF PURSUIT.

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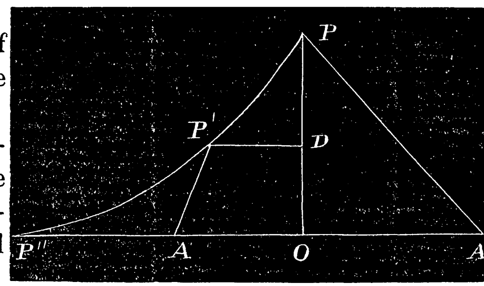
Problem. A boat starts from a given point in the bank of a straight canal of uniform current, and is propelled continually toward a movable object, A, on the opposite bank. To find the path of the boat.

It will be seen that this is an extension of the problem discussed in the May No. of the ANALYST, page 89 et seq. It is here proposed to give a few of the results of an investigation of the above problem leaving their demonstration to be supplied by those of your readers who are interested in the subject.



Let P be the starting point of the boat; and A, that of the moveable object, $\mathfrak{A}$.

Put PO, the width of the canal $= b$; $PA = a$; and angle $PAO = \beta$. All these are supposed to be given and are reg'ed as constants.

Let P' be the position of the boat at any instant, and A' the corresponding position of the object, $\mathfrak{A}$. Put $P'A' = z$; and angle $P'A'P'' = \varphi$. Take O as the origin of rectangular coordinates, and make $OD = x$, and $P'D = y$.

Designate the velocity of the boat relative to the water by u; the velocity of the current by v; and that of the object, $\mathfrak{A}$, by w. Consider motion positive in the direction AP''. Also put $[(w-v)\div u] = m$; and $v \div u = n$. Then it may be shown that $x^m \tan \frac{1}{2}\varphi$ is constant, and therefore equal to $b^m \tan \frac{1}{2}\beta$. Make this equal to c^m, where c is the value of z when it is perpendicular to AP''.

From the above we find the equation of the curve when $m < 1$, to be,

$$y = \frac{(1-n)x^{1+m}}{2c^m(1+m)} - \frac{(1+n)c^m x^{1-m}}{2(1-m)} + \frac{a[m+n-(1+mn)\cos\beta]}{1-m^2}. \qquad (1)$$

When $m > 1$ we must take $m-1$ instead of $1-m$. When $m = 1$, we have,

$$y = \frac{c(1+n)}{2}\log\frac{b}{x} - \frac{1-n}{4c}\left(b^2 - x^2\right); \qquad (2)$$

and
$$c = b\tan\tfrac{1}{2}\beta = a(1-\cos\beta).$$

The above formulas are true for all finite values of u, v and w, except when $v = w$ (and $\therefore m = 0$). In this case the path of the boat is a

straight line whose equation is, supposing $u > v$,

$$y = \frac{b-x}{\sin\beta}\left(n - \cos\beta\right).$$

When $v = 0$ (and $\therefore\ n = 0$), equations (1) and (2) represent the ordinary curve of pursuit.

If we make $m = \frac{1}{2}$ and $n = 1$ ($\therefore\ u = v = \frac{2}{3}w$), the path becomes a parabola whose focal distance is equal to $c = b \tan \frac{1}{2}\beta$.

As a final example, suppose $w = 0$; therefore $m = -n$. That is, the boat is directed toward a *fixed object*. Equation (1) reduces to

$$y = \frac{c^m x^{1-m}}{2} - \frac{x^{1+m}}{2c^m} - a\cos\beta;$$

and the expression for c becomes

$$c^m = b^m \cot \tfrac{1}{2}\beta.$$

If in the last equation we make $m = 1$, we shall have

$$y = \frac{a(1-\cos\beta)}{2} - \frac{x^2}{2c};$$

the equation of a parabola whose axis is AP'', and whose focal dist. is $\frac{1}{2}c$.

If $t =$ the time from the starting point, we shall have,

$$ut = \frac{a(1 - m\cos\beta)}{1-m^2} - \frac{c^m x^{1-m}}{2(1-m)} - \frac{x^{1+m}}{2c^m(1+m)}.$$

This equation is true for all finite values of m, positive or negative. When $x = 0$, and $m < 1$, we have the whole time of crossing

$$T = \frac{a(1 - m\cos\beta)}{u(1-m^2)}.$$

If $\cos\beta = m$, we have $T = a \div u$. That is, if the boat, at starting, be directed toward an object at A, so that the angle $PAO = \cos^{-1} m$; then, as the object, A, moves from A to P'', the boat will describe the curve $PP'P''$ in the same time that would be required to move from P to A along the line PA with the same velocity, if there were no current.

MR. GLAISHER'S ENUMERATION OF PRIMES.

COMMUNICATED BY PROF. JOHNSON.

In the Report for 1879, of the committee on mathematical Tables to the British Association, Mr. J. W. L. Glaisher extends his enumeration of primes over the fourth million, by means of Mr. James Glaisher's factor table for this million, which has just been printed. The results of the enumeration of primes are given in the same form as those referred to at p. 7, Vol.